Révision pour les identités

1. a)

$$\frac{\sin^2 x\ (1+\tan^2 x)}{\cos^2 x} \cdot \frac{\cot an^2 x}{\operatorname{cosec}^2 x}$$

$$\frac{\sin^2 x\ (\sec^2 x)}{\cos^2 x} \cdot \frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}}$$

$$\frac{\sin^2 x\ \left(\frac{1}{\cos^2 x}\right)}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1}$$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\cancel{\cos^2 x}} \cdot \frac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \frac{\sin^2 x}{1} = \tan^2 x$$

b)

$$\cancel{\sin^2 A} \cdot \frac{\cos^2 A}{\cancel{\sin^2 A}} + \cos^2 A\ (\tan^2 A)$$

$$\cos^2 A + \cancel{\cos^2 A} \cdot \frac{\sin^2 A}{\cancel{\cos^2 A}}$$

$$\cos^2 A + \sin^2 A = 1$$

c)

$$\cot an\, x\ (\cos x + \tan x \sin x)$$

$$\frac{\cos x}{\sin x}\left(\cos x + \frac{\sin x}{\cos x} \cdot \sin x\right)$$

$$\frac{\cos x}{\sin x}\left(\frac{\cos^2 x + \sin^2 x}{\cos x}\right)$$

$$\frac{\cancel{\cos x}}{\sin x}\left(\frac{1}{\cancel{\cos x}}\right) = \frac{1}{\sin x} = \operatorname{cosec} x$$

d) $\dfrac{\sin x}{\operatorname{cosec} x} + \dfrac{\cos x}{\sec x}$

$\dfrac{\sin x}{\frac{1}{\sin x}} + \dfrac{\cos x}{\frac{1}{\cos x}}$

$\sin x \cdot \dfrac{\sin x}{1} + \cos x \cdot \dfrac{\cos x}{1}$

$\sin^2 x + \cos^2 x = 1$

2. $\cos x \tan x + 2\sin^2 x - 1 = 0$

$\cancel{\cos x} \cdot \dfrac{\sin x}{\cancel{\cos x}} + 2\sin^2 x - 1 = 0$

$\sin x + 2\sin^2 x - 1 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot -1}}{4} \Rightarrow \dfrac{-1+3}{4}$ et $\dfrac{-1-3}{4}$

0,5 et -1

$\sin x = 0{,}5$ ou $\sin x = -1$

$\theta 1 = \sin^{-1}(0{,}5) = 30$ ou $\frac{\pi}{6}$ $\quad$ $\theta 1 = \sin^{-1}(-1) = -90$ ou 270 ou $\frac{3\pi}{4}$

$\theta 2 = 180 - 30 = 150$ ou $\frac{5\pi}{6}$ $\quad$ $\theta 2 = 180 - 270 = -90$ ou 270

$\frac{\pi}{6} + 2n\pi$

$\frac{5\pi}{6} + 2n\pi$

$\frac{3\pi}{4} + 2n\pi$

$\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{4} \right\}$

3. a) $$\frac{1}{\cos x} - \frac{\cos x}{1+\sin x} = \tan x$$

$$\frac{1+\sin x - \cos^2 x}{\cos x\,(1+\sin x)} = \tan x$$

$$\frac{\sin^2 x + \sin x}{\cos x\,(1+\sin x)} = \tan x$$

$$\frac{\sin x\,\cancel{(\sin x + 1)}}{\cos x\,\cancel{(1+\sin x)}} = \tan x$$

$$\frac{\sin x}{\cos x} = \tan x$$

$$\tan x = \tan x$$

Resoudre

b) $$2\cos^2 x + \sin x - 1 = 0$$

$$2\,(1-\sin^2 x) + \sin x - 1 = 0$$

$$2 - 2\sin^2 x + \sin x - 1 = 0$$

$$-2\sin^2 x + \sin x + 1 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-1 \pm \sqrt{1^2 - 4 . -2 . 1}}{-4}$$

$$\frac{-1+3}{-4} \qquad \frac{-1-3}{-4}$$

$$-0{,}5 \qquad 1$$

$\sin x = -0{,}5$ $\qquad$ $\sin x = 1$

$\theta_1 = \sin^{-1} 0{,}5 = -30$ ou 330 ou $\frac{11\pi}{6}$ $\qquad$ $\theta_1 = \sin^{-1}(1) = 90$ ou $\frac{\pi}{2}$

$\theta_2 = 180-30 = 210°$ ou $\frac{7\pi}{3}$ $\qquad$ $\theta_2 = 180-90 = 90$ ou $\frac{\pi}{2}$

c) $\dfrac{\tan^2 x}{1+\tan^2 x} \cdot \dfrac{1+\cot^2 x}{\cot^2 x} = \tan^2 x$

$\dfrac{\tan^2 x}{\sec^2 x} \cdot \dfrac{\operatorname{cosec}^2 x}{\cot^2 x} = \tan^2 x$

$\dfrac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} \cdot \dfrac{\frac{1}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}} = \tan^2 x$

$\dfrac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x} \cdot \dfrac{1}{\cancel{\sin^2 x}} \cdot \dfrac{\cancel{\sin^2 x}}{\cos^2 x} = \tan^2 x$

$\dfrac{\sin^2 x}{\cos^2 x} = \tan^2 x$

$\tan^2 x = \tan^2 x$

d) $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = 2\operatorname{cosec} x$

$\dfrac{\sin x \cdot \sin x + (1+\cos x)(1+\cos x)}{(1+\cos x)(\sin x)} = 2\operatorname{cosec} x$

$\sin^2 x + 1 + \cos x + \cos x + \cos^2 x = \dfrac{2}{\cancel{\sin x}} \cdot (1+\cos x)\cancel{\sin x}$

$1 + 1 + 2\cos x = 2 + 2\cos x$

$2 + 2\cos x = 2 + 2\cos x$

e) $\dfrac{2\tan x}{1-\tan^2 x} = \tan 2x$

$\dfrac{2\tan x}{1-\frac{\sin^2 x}{\cos^2 x}} = \tan 2x$

$\dfrac{2\tan x}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} = \tan 2x$

$\dfrac{2\sin x}{\cancel{\cos x}} \cdot \dfrac{\overset{\cos x}{\cancel{\cos^2 x}}}{\cos^2 - \sin^2 x} = \tan 2x$

$\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \tan 2x$

$\dfrac{\sin 2x}{\cos 2x} = \tan 2x$

$\tan 2x = \tan 2x$

f) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \sec x$$

$$\frac{\sin x(1+\sin x) + \cos x \cdot \cos x}{\cos x(1+\sin x)} = \sec x$$

$$\frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1+\sin x)} = \sec x$$

$$\frac{\cancel{\sin x + 1}}{\cos x\,(\cancel{1+\sin x})} = \sec x$$

$$\frac{1}{\cos x} = \sec x$$

$$\sec x = \sec x$$

g) $\cot A\,(\sec A - \cos A) = \sin A$

$$\frac{\cos A}{\sin A}\left(\frac{1}{\cos A} - \cos A\right) = \sin A$$

$$\frac{1}{\sin A} - \frac{\cos^2 A}{\sin A} = \sin A$$

$$\frac{1-\cos^2 A}{\sin A} = \sin A$$

$$\frac{\sin^2 A}{\sin A} = \sin A$$

$$\sin A = \sin A$$

h) $(1-\sin^4 x)(1+\tan^2 x) - \dfrac{1}{\operatorname{cosec}^2 x - \cot^2 x} = \sin^2 x$

$(1+\sin^2 x)(1-\sin^2 x)\ \sec^2 x - \dfrac{1}{1} = \sin^2 x$

$(1+\sin^2 x)\cdot \cancel{\cos^2 x}\cdot \dfrac{1}{\cancel{\cos^2 x}} - 1 = \sin^2 x$

$1+\sin^2 x - 1 = \sin^2 x$

$\sin^2 x = \sin^2 x$

École secondaire le Carrefour
Maths SN5

Nom : ________________
Gr : ________

Révision pour les identités

1. Simplifier :

   a) $\frac{\sin^2 x\,(1+\tan^2 x)}{\cos^2 x} \cdot \frac{\cot an^2 x}{\operatorname{cosec}^2 x}$

   b) $\sin^2 A \cot^2 A + \cos^2 A\,(\sec^2 A - 1)$

   c) cotanx (cosx + tanx sinx)

   d) $\frac{\sin x}{\operatorname{cosec} x} + \frac{\cos x}{\sec x}$

2. Résoudre l'équation ci-dessous dans l'intervalle $[0,2\pi]$
   $\cos x \tan x + 2\sin^2 x = 1$

3. Démontrer :

   a) $\frac{1}{\cos x} - \frac{\cos x}{1+\sin x} = \tan x$

   b) $2\cos^2 x + \sin x - 1 = 0$ (Resoudre)

   c) $\frac{\tan^2 x}{1+\tan^2 x} \cdot \frac{1+\cot^2 x}{\cot^2 x} = \tan^2 x$

   d) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\operatorname{cosec} x$

   e) $\frac{2\tan x}{1-\tan^2 x} = \tan 2x$

   f) $\tan x + \frac{\cos x}{1+\sin x} = \sec x$

   g) $\cot A\,(\sec A - \cos A) = \sin A$

   h) $(1 - \sin^4 x)\,(1 + \tan^2 x) - \frac{1}{\operatorname{cosec}^2 x - \cot^2 x} = \sin^2 x$